



SB-2733

M. Sc. (Part - I) (Sem. II) Examination

April / May - 2011

Mathematics

(Functional Analysis)

Time : 3 Hours]

[Total Marks : 70

Instructions :

(1)

नीचे दशांशवले निशानीवाणी विगतो उत्तरवडी पर अवश्य लभवी. Fillup strictly the details of signs on your answer book.	Seat No. :
Name of the Examination :	<input type="text"/>
<input type="text" value="M.Sc. (Part -1)(Sem. 2)"/>	<input type="text"/>
Name of the Subject :	<input type="text"/>
<input type="text" value="Mathematics"/>	<input type="text"/>
Subject Code No. : <input type="text" value="2"/> <input type="text" value="7"/> <input type="text" value="3"/> <input type="text" value="3"/>	Section No. (1, 2,.....) : <input type="text" value="Nil"/>
Student's Signature	

- (2) Attempt all questions.
(3) Follow the usual notations and conventions.
(4) Figures on the right indicate full marks.

1 Attempt any two : 14

- (1) Prove that the function space $C[a, b]$ is complete where $[a, b]$ is any given closed interval on \mathbb{R} .
(2) Prove that a mapping $T : X \rightarrow Y$ of a metric space (X, d) into a metric space (Y, d') is continuous at a point $x_0 \in X$ if and only if $x_n \rightarrow x_0 \Rightarrow Tx_n \rightarrow Tx_0$.
(3) Prove that on a finite dimensional vector space X , any norm $\|\cdot\|$ is equivalent to any other norm $\|\cdot\|_0$.

2 Attempt any two : 14

- (1) Prove that every finite dimensional subspace Y of a normed space X is complete.
(2) Let $T : D(T) \rightarrow Y$ be a linear operator, where $D(T) \subset X$ and X, Y are normed spaces. Prove that :
(a) T is continuous if and only if T is bounded.
(b) If T is continuous at a single point it is continuous.
(3) Prove that the dual space of \mathbb{R}^n is \mathbb{R}^n .

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[Contd...

3 Attempt any two : 14

- (1) Let and prove Riesz's lemma.
- (2) Let $T : D(T) \rightarrow Y$ be a bounded linear operator where $D(T)$ lies in a normed space X and Y be a Banach space. Prove that T has an extension $\tilde{T} : D(T) \rightarrow Y$ where \tilde{T} is a bounded linear operator with the norm $\|\tilde{T}\| = \|T\|$.
- (3) Prove that a finite dimensional vector space is algebraically reflexive.

4 Attempt any two : 14

- (1) Let X be an inner product space and Y be its complete subspace and $x \in X$. Prove that there exists unique $y \in Y$ such that $(x-y) \perp Y$.
- (2) Let Y be any closed subspace of a Hilbert space H . Prove that $H = Y \oplus Y^\perp$.
- (3) Let $T : H_1 \rightarrow H_2$ be the bounded linear operator. Prove that there exists a bounded linear operator $T^* : H_2 \rightarrow H_1$ such that $\langle T x, y \rangle = \langle x, T^* y \rangle, \forall x \in H_1, y \in H_2$ and $\|T^*\| = \|T\|$.

5 Attempt any two : 14

- (1) For any subset $M \neq \emptyset$ of a Hilbert space H , prove that the span of M is dense in H if and only if $M^\perp = \{0\}$.
- (2) Let (e_k) be an orthonormal sequence in an inner product space X . Prove that for every $x \in X, \sum_{k=1}^{\infty} |\langle x, e_k \rangle|^2 \leq \|x\|^2$.
- (3) Let H_1, H_2 be Hilbert spaces, $S : H_1 \rightarrow H_2$ and $T : H_1 \rightarrow H_2$ bounded linear operators and α any scalar. Prove that :
 - (a) $\langle T^* y, x \rangle = \langle y, T x \rangle \quad (x \in H_1, y \in H_2)$
 - (b) $(S+T)^* = S^* + T^*$
 - (c) $(\alpha T)^* = \bar{\alpha} T^*$
 - (d) $(T^*)^* = T$.